## Exercise 176

A total of $250,000 \mathrm{~m}^{2}$ of land is needed to build a nuclear power plant. Suppose it is decided that the area on which the power plant is to be built should be circular.
a. Find the radius of the circular land area.
b. If the land area is to form a $45^{\circ}$ sector of a circle instead of a whole circle, find the length of the curved side.

## Solution

## Part (a)

The area of a circle is given by

$$
A=\pi r^{2}
$$

where $r$ is the radius and $\pi \approx 3.14$. Solve the equation for $r$ by dividing both sides by $\pi$

$$
\frac{A}{\pi}=r^{2}
$$

and then taking the square root of both sides.

$$
\sqrt{\frac{A}{\pi}}=r
$$

Therefore, plugging in the area of the plant,

$$
r=\sqrt{\frac{250,000 \mathrm{~m}^{2}}{\pi}} \approx 280 \mathrm{~m}
$$

## Part (b)

The area of a sector of a circle is given by

$$
A=\frac{1}{2} r^{2} \theta,
$$

where $r$ is the radius and $\theta$ is the angle in radians. Solve this equation for $r$.

$$
\begin{aligned}
& 2 A=r^{2} \theta \\
& \frac{2 A}{\theta}=r^{2} \\
& \sqrt{\frac{2 A}{\theta}}=r
\end{aligned}
$$

The formula for the length of the curved side of this sector is

$$
s=r \theta
$$

Therefore, substituting the formula for $r$,

$$
\begin{aligned}
s & =\left(\sqrt{\frac{2 A}{\theta}}\right) \theta \\
& =\sqrt{2 A \theta} \\
& =\sqrt{2\left(250,000 \mathrm{~m}^{2}\right)\left(45^{\circ} \times \frac{\pi}{180^{\circ}}\right)} \\
& \approx 630 \mathrm{~m} .
\end{aligned}
$$

